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MATHEMATICAL MODEL OF THE UNSTEADY MOTION OF A SHAFT IN A HYDRODYNAMIC PLAIN BEARING

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An asymptotic solution is obtained that describes the unsteady motion of a shaft in a cylindrical plain bearing with hydrodynamic lubrication in the case of a constant external load. Oscillatory modes of transition to a steady-state position of the shaft for various values of the external load are considered. The characteristic time of velocity relaxation to the quasiequilibrium values determined from the inertialess approximation equations is obtained. Oscillation frequencies and amplitudes, shaft paths, and oscillation decay times are determined. The effect of a thin elastic liner on the characteristics of the transient process is explored.

Key words: *hydrodynamic bearing, unsteady motion, asymptotic solution.*

Introduction. Hydrodynamic plain bearings are important structural members of modern machines. They support rotating shafts coated with a thin lubricant layer. The motion of the thin oil layer of variable thickness between the rotating shaft of a machine and a fixed bearing pad leads to a considerable increase in the clearance pressure and gives rise to a supporting force. For such bearings, existing computational methods are based on integration of the Reynolds equations [1] obtained from the Navier–Stokes equations in the Stokes approximation. The steady-state hydrodynamic lubrication regime in cylindrical bearings has been adequately explored. In particular, in [2, 3], the basic steady-state regimes were considered: (a) hydrodynamic contact of rigid cylinders of infinite and finite lengths; (b) elastohydrodynamic contact of cylinders.

The unsteady dynamics and stability of bearings have been studied less extensively, but they play an important role in transient phenomena. Results from numerical modeling of some unsteady regimes of a plain bearing in the rigid surface approximation are presented in [4, 5].

The objective of the present study is an asymptotic analysis of the unsteady motion of a shaft in the clearance of a cylindrical plain bearing taking into account a thin elastic liner under a constant external load. The motion paths of the shaft are considered, and the amplitudes, frequencies, and oscillation decay times for various values of the external load are determined.

1. Formulation of the Problem. System of Equations for the Lubricant Layer. The motion of oil films between surfaces is usually described by the Reynolds equations [2, 3]

$$\operatorname{div}\Big(\frac{h^3}{12\mu}\,\nabla P\Big) = \operatorname{div}(\boldsymbol{U}h) + \frac{\partial h}{\partial t}, \tag{1}$$

where div is a two-dimensional divergence operator on a specified boundary surface, h is the film thickness, P is the pressure, μ is the viscosity, and $\boldsymbol{U} = (\boldsymbol{U}_1 + \boldsymbol{U}_2)/2$ ($\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are specified velocities on the surfaces bordering the film).

For the lubricant film between infinite cylinders, the derivatives along the axis of the cylinders are equal to zero and the Reynolds equation (1) takes the simpler form

$$\frac{\partial}{\partial s}\Big(\frac{h^3}{12\mu}\,\frac{\partial P}{\partial s}\Big) = \frac{\partial}{\partial s}(Uh) + \frac{\partial h}{\partial t}, \tag{2}$$

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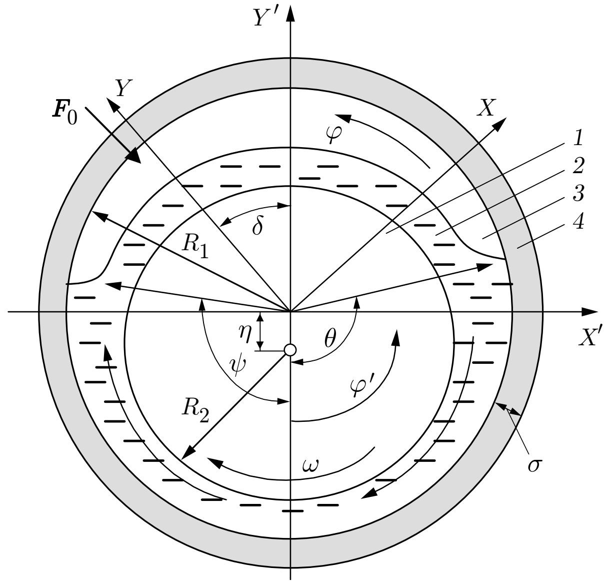

Fig. 1. Cross section of a hydrodynamic plain bearing: 1) shaft; 2) liquid lubricant layer; 3) clearance; 4) elastic liner.

where s is the distance along the streamlined contour in the plane of the flow and U is the half-sum of the velocities of the cylindrical surfaces. Considering the problem of the motion of the lubricant layer in the clearance between an internal cylinder of radius R_2 rotating at angular velocity ω and the fixed external cylinder of radius R_1 (Fig. 1), we convert from the linear coordinate s to an angular variable φ. We assume that the shaft rotates clockwise and the angular variable is reckoned counterclockwise. In this case,

$$\frac{\partial}{\partial s} = -\frac{\partial}{R_1 \partial \varphi}, \qquad U = \frac{\omega R_2}{2}. \tag{3}$$

Let us introduce a moving coordinate system (X', Y') and a fixed coordinate system (X, Y) (Fig. 1). At each time, the Y' axis of the moving coordinate system is oppositely directed to the displacement of the center of the shaft. In this case, the Y axis of the fixed system is opposite the vector of the external constant force.

The contour of the shaft in the plane (X, Y) is a circle with a displaced center and is described in polar coordinates by the equation

$$r = \eta \cos \varphi' + \sqrt{R_2^2 - \eta^2 \sin^2 \varphi'} = R_2 + \eta \cos \varphi' + R_2 O(\eta^2 / R_2^2), \tag{4}$$

where r is the distance from the point of intersection of the X and Y axes, η is the displacement of the shaft axis, φ' is the angle reckoned counterclockwise from the negative Y' semiaxis (Fig. 1). Using (4) and taking into account the elastic deformation of the liner, we find the thickness of the clearance between the cylindrical surfaces

$$h = R_1 - r + \xi = R_1 - R_2 - \eta \cos \varphi' + \xi + R_2 O(\eta^2 / R_2^2), \tag{5}$$

where the variable ξ characterizes the radial elastic displacements of the liner surface. The problem of the deformation of the surface of a thin liner fixed in an absolutely rigid case contains a small parameter equal to the ratio of the liner thickness σ to the curvature radius R_1. As shown in [3], in the first-order expansion in the parameter σ / R_1, the fluid film pressure is in direct proportion to the strain of the liner surface:

$$\xi = CP, \qquad C = \sigma(1 + \nu)(1 - 2\nu)/(E(1 - \nu)). \tag{6}$$

Here ν and E are Poisson's constant and Young's modulus for the material of the elastic layer. In view of equalities (3), (5), and (6), Eq. (2) becomes

$$\frac{\partial}{\partial \varphi}\Big(h^3 \frac{\partial P}{\partial \varphi}\Big) = -6\mu\omega R_1 R_2 \frac{\partial h}{\partial \varphi} + 12\mu R_1^2 \frac{\partial h}{\partial t}. \tag{7}$$

In Eq. (7), the layer thicknesses h depends on the azimuthal angle and pressure (in the presence of an elastic liner):

$$h = \Delta[1 - \lambda\cos\varphi' + CP + \lambda^2 O(\Delta/R_2)], \qquad \lambda = \eta/\Delta, \qquad \varphi' = \varphi + \delta + \pi/2. \tag{8}$$

Here $\Delta = R_1 - R_2$; the angle δ characterizes the direction of displacement of the shaft relative to the fixed coordinate system (X, Y) (Fig. 1).

If the clearance is incompletely filled with the lubricant, the input boundary conditions ($\varphi' = \theta$) and output boundary conditions ($\varphi' = -\psi$) for the lubricant layer are written as

$$\varphi' = \theta\colon \quad P = 0, \qquad \varphi' = -\psi\colon \quad P = 0, \quad \frac{\partial P(\varphi')}{\partial \varphi'} = 0.$$

If the clearance is completely filled with the lubricant, the angles θ and ψ are linked by the simple relation $\theta = 2\pi - \psi$.

The external load on the shaft is compensated for by the force produced by the overpressure in the liquid lubricant layer. A unit surface area is acted upon by a force equal to the pressure and directed normal to the surface. The components of the complete force vector per unit length of the shaft are evaluated by integration of the pressure distribution function along the surface of the shaft:

$$W'_x = \int\limits_{-\psi}^{\theta} P(\varphi') N_x \, d\Sigma, \qquad W'_y = \int\limits_{-\psi}^{\theta} P(\varphi') N_y \, d\Sigma. \tag{9}$$

Here W'_x and W'_y are the projections of the resulting pressure force per unit length of the shaft onto the X' and Y' axes, respectively, N_x and N_y are the components of the normal vector to the shaft surface, and $d\Sigma$ is the differential of the arc length of the shaft contour. Using Eq. (4), we obtain the following expressions for the differential $d\Sigma$ and the components of the normal vector to the shaft surface in polar coordinates:

$$d\Sigma = d\varphi' \sqrt{r^2 + \Big(\frac{dr}{d\varphi'}\Big)^2} = d\varphi' \sqrt{R_2^2 + 2R_2\eta\cos\varphi' + \eta^2} = R_2\Big[1 + O\Big(\frac{\eta}{R_2}\Big)\Big]\, d\varphi'; \tag{10}$$

$$N_r = 1 + O(\eta^2/R_2^2), \qquad N_\varphi = -\eta\sin\varphi'[1 + O(\eta/R_2)]. \tag{11}$$

From the known components (11), we determine the projections of the normal vector onto the X' and Y' axis:

$$N'_x = -\sin\varphi' + O(\eta/R_2), \qquad N'_y = \cos\varphi' + O(\eta/R_2). \tag{12}$$

In view of equalities (10) and (12), expressions (9) become

$$W'_x = -R_2 \int\limits_{-\psi}^{\theta} P(\varphi')\Big[\sin\varphi' + O\Big(\frac{\eta}{R_2}\Big)\Big]\, d\varphi', \quad W'_y = R_2 \int\limits_{-\psi}^{\theta} P(\varphi')\Big[\cos\varphi' + O\Big(\frac{\eta}{R_2}\Big)\Big]\, d\varphi'. \tag{13}$$

To solve the problem, it is convenient to convert to dimensionless variables:

$$P = 6\mu\omega R_1^2 q/\Delta^2, \qquad t = 2t'/\omega, \qquad h = H\Delta. \tag{14}$$

Ignoring small terms on the order of $O(\Delta/R_2)$ in expression (8) and taking into account (14), we obtain

$$H = 1 - \lambda\cos\varphi' + \alpha q, \qquad \alpha = \frac{6(1+\nu)(1-2\nu)}{1-\nu}\, \frac{\mu\omega\sigma R_1^2}{E\Delta^3}.$$

In the unsteady regime, the parameters λ and δ, characterizing the displacement of the shaft axis, depend on time. Using the relation $\varphi' = \varphi + \delta + \pi/2$, we find the time derivative of the layer thickness:

$$\Big(\frac{\partial H}{\partial t'}\Big)_\varphi = -\frac{d\lambda}{dt}\, \cos\varphi' + \lambda\, \frac{d\delta}{dt}\, \sin\varphi' + \alpha\, \frac{\partial q}{\partial t}. \tag{15}$$

Making the change of variables (14), using equality (15), and assuming $R_1/R_2 \approx 1$, we bring Eq. (7) to the form

$$\frac{\partial}{\partial\varphi'}\Big(H^3\, \frac{\partial q}{\partial\varphi'}\Big) = -\lambda\sin\varphi' - \alpha\, \frac{\partial q}{\partial\varphi'} - \frac{d\lambda}{dt}\, \cos\varphi' + \lambda\, \frac{d\delta}{dt}\, \sin\varphi' + \alpha\, \frac{\partial q}{\partial t'}. \tag{16}$$

2. Equations of Shaft Oscillations. We consider shaft oscillations that arise under a constant external load. A force directed to the shaft axis acts in the neighborhood of each point on the shaft surface. The components of the resultant force in the moving coordinate system (X', Y') are defined by formulas (13). Then, in the fixed coordinate system (X, Y), the components of the resultant pressure force are determined from the formulas of orthogonal transformation (rotation through an angle δ)

$$W_x = W_x' \cos\delta + W_y' \sin\delta, \qquad W_y = -W_x' \sin\delta + W_y' \cos\delta. \tag{17}$$

Using equalities (17) and taking into account the constant external force, we write the second Newton's laws for the shaft in the projections onto the X and Y axes:

$$m\,\frac{d^2X}{dt^2} = W_x' \cos\delta + W_y' \sin\delta, \qquad m\,\frac{d^2Y}{dt^2} = -W_x' \sin\delta + W_y' \cos\delta - F_0. \tag{18}$$

Here m and F_0 are the mass of the shaft and the external force per unit length of the shaft. Converting to dimensionless pressure [see (14)] and ignoring small terms on the order of η/R_2 and Δ/R_2 in formulas (13), we obtain the expressions

$$W_x' = -\frac{6\mu\omega R_1^3}{\Delta^2} \int\limits_{-\psi}^{\theta} q(\varphi) \sin\varphi \, d\varphi, \qquad W_y' = \frac{6\mu\omega R_1^3}{\Delta^2} \int\limits_{-\psi}^{\theta} q(\varphi) \cos\varphi \, d\varphi.$$

The coefficients W_x' and W_y' depend on the parameters λ and δ, characterizing the shaft position, and on the velocities $d\lambda/dt$ and $d\delta/dt$. Following [4], we linearize the dependences of the factors on the velocities and write them as follows:

$$W_x' = \frac{6\mu\omega R_1^3}{\Delta^2}\Big(c_\delta\lambda + d_\lambda\,\frac{d\lambda}{dt} + d_\delta\lambda\,\frac{d\delta}{dt}\Big), \qquad W_y' = \frac{6\mu\omega R_1^3}{\Delta^2}\Big(c_\lambda\lambda + b_\lambda\,\frac{d\lambda}{dt} + b_\delta\lambda\,\frac{d\delta}{dt}\Big). \tag{19}$$

The coefficients on the right sides of equalities (19) are determined by solving the Reynolds equations. Calculations show that the factors b_δ and d_λ are small compared with b_λ and d_δ. The latter will be called the radial and azimuthal damping factors, respectively. The parameters c_δ and c_λ will be called the rigidity factors.

The displacements of the center of the shaft along the X and Y axes are linked to the relative radial transition λ and the azimuthal angle δ by the relations

$$X = -\Delta\lambda\sin\delta, \qquad Y = -\Delta\lambda\cos\delta.$$

Differentiating these equalities with respect to time, we obtain

$$\begin{aligned}
\frac{d^2X}{dt^2} &= \Delta\Big[-\frac{d^2\lambda}{dt^2}\sin\delta - 2\,\frac{d\lambda}{dt}\,\frac{d\delta}{dt}\cos\delta - \frac{d^2\delta}{dt^2}\,\lambda\cos\delta + \Big(\frac{d\delta}{dt}\Big)^2\lambda\sin\delta\Big],\\
\frac{d^2Y}{dt^2} &= \Delta\Big[-\frac{d^2\lambda}{dt^2}\cos\delta + 2\,\frac{d\lambda}{dt}\,\frac{d\delta}{dt}\sin\delta + \frac{d^2\delta}{dt^2}\,\lambda\sin\delta + \Big(\frac{d\delta}{dt}\Big)^2\lambda\cos\delta\Big].
\end{aligned} \tag{20}$$

We multiply the first and second Eqs. (18) by $\sin\delta$ and $\cos\delta$, respectively, and sum the equalities. As a result, using expressions (19) and (20), we obtain the following equation for the azimuthal acceleration of the center of the shaft:

$$\varepsilon\Big[\frac{d^2\lambda}{dt^2} - \lambda\Big(\frac{d\delta}{dt}\Big)^2\Big] = -c_\lambda\lambda - b_\lambda\,\frac{d\lambda}{dt} + F\cos\delta. \tag{21}$$

Here F is the dimensionless external force linked to the dimensional force F_0 by the relation $F_0 = 6\mu\omega R_1^3 F/\Delta^2$; $\varepsilon = m\Delta^3\omega/(6\mu R_1^3) \ll 1$ is a dimensionless small parameter.

Similarly, multiplying the first and second of Eqs. (18) by $-\lambda\cos\delta$ and $\lambda\sin\delta$, using (19) and (20), and summing the equalities, we obtain the following equation for the azimuthal acceleration of the center of the shaft:

$$\varepsilon\,\frac{d}{dt}\Big(\lambda^2\,\frac{d\delta}{dt}\Big) = -c_\delta\lambda^2 - d_\delta\lambda^2\,\frac{d\delta}{dt} - F\lambda\sin\delta. \tag{22}$$

Equations (21) and (22) contain the singular small parameter ε at higher derivatives. According to the known method of [6], the solution of such a system is represented as the sum of two asymptotic series

$$\lambda = \lambda_r(t, \varepsilon) + \lambda_s(t/\varepsilon, \varepsilon), \qquad \delta = \delta_r(t, \varepsilon) + \delta_s(t/\varepsilon, \varepsilon), \tag{23}$$

where the first terms are regular parts of the asymptotic relations

$$\begin{aligned}
\lambda_r(t, \varepsilon) &= \lambda_r^{(0)}(t) + \varepsilon\lambda_r^{(1)}(t) + \ldots + \varepsilon^n\lambda_r^{(n)}(t) + \ldots,\\
\delta_r(t, \varepsilon) &= \delta_r^{(0)}(t) + \varepsilon\delta_r^{(1)}(t) + \ldots + \varepsilon^n\delta_r^{(n)}(t) + \ldots,
\end{aligned} \tag{24}$$

and the second terms contains boundary functions that describe rapid motions

$$\lambda_s(t/\varepsilon,\varepsilon) = \lambda_s^{(0)}(t/\varepsilon) + \varepsilon\lambda_s^{(1)}(t/\varepsilon) + \ldots + \varepsilon^n\lambda_s^{(n)}(t/\varepsilon) + \ldots,$$
$$\delta_s(t/\varepsilon,\varepsilon) = \delta_s^{(0)}(t/\varepsilon) + \varepsilon\delta_s^{(1)}(t/\varepsilon) + \ldots + \varepsilon^n\delta_s^{(1)}(t/\varepsilon) + \ldots\ . \tag{25}$$

The boundary functions should tend to zero as their argument tends to infinity:

$$\lambda_s^{(m)}(\infty) = 0, \qquad \delta_s^{(m)}(\infty) = 0, \qquad m = 0, 1, 2, \ldots\ .$$

Substituting a solution of the form (23)–(25) into system (21), (22) and equating the coefficients at identical powers of the small parameter on both sides of the equalities, we obtain equations that define the coefficients of the asymptotic series (24) and (25). In this case, it is necessary to equate the coefficients dependent on t and the coefficients dependent on t/ε. Thus, as a zero approximation we have the system

$$c_\delta\lambda_r^{(0)} + d_\delta\lambda_r^{(0)}\,\frac{d\delta_r^{(0)}}{dt} + F\sin\delta_r^{(0)} = 0, \qquad -c_\lambda\lambda_r^{(0)} - b_\lambda\,\frac{d\lambda_r^{(0)}}{dt} + F\cos\delta_r^{(0)} = 0.$$

We reduce this system to normal form

$$\frac{d\delta_r^{(0)}}{dt} = -\frac{1}{d_\delta\lambda_r^{(0)}}\,(c_\delta\lambda_r^{(0)} + F\sin\delta_r^{(0)}), \qquad \frac{d\lambda_r^{(0)}}{dt} = \frac{1}{b_\lambda}\,(-c_\lambda\lambda_r^{(0)} + F\cos\delta_r^{(0)}). \tag{26}$$

Eliminating time, we obtain the following first-order equation that defines the motion path:

$$\frac{d\delta_r^{(0)}}{d\lambda_r^{(0)}} = -\frac{b_\lambda(c_\delta\lambda_r^{(0)} + F\sin\delta_r^{(0)})}{d_\delta\lambda_r^{(0)}(-c_\lambda\lambda_r^{(0)} + F\cos\delta_r^{(0)})}.$$

The coefficient c_λ depends on the properties of the liner (parameter α). In the absence of a liner ($\alpha = 0$), we have $c_\lambda = 0$. This condition reduces the equation to a simple first-order equation for $\sin\delta$:

$$\frac{d\sin\delta_r^{(0)}}{d\lambda_r^{(0)}} = -\frac{b_\lambda(c_\delta\lambda_r^{(0)} + F\sin\delta_r^{(0)})}{d_\delta\lambda_r^{(0)}F}. \tag{27}$$

The general solution of the linear nonuniform equation (27) has the form

$$\sin\delta_r^{(0)} = \sin\delta_0\exp\Big(-\int\limits_{\lambda_0}^{\lambda}\frac{b_\lambda}{\lambda' d_\delta}\,d\lambda'\Big) - \frac{1}{F}\int\limits_{\lambda_0}^{\lambda}\frac{c_\delta b_\lambda}{d_\delta}\exp\Big(\int\limits_{\lambda}^{\lambda'}\frac{b_\lambda}{\lambda'' d_\delta}\,d\lambda''\Big)\,d\lambda',$$

where λ_0 and δ_0 are the initial the coordinates of the center of the shaft. Applying a Taylor expansion to the right sides of (26) in the neighborhood of the point of rest, we obtain the following system of first-approximation equations

$$\frac{d\delta_r^{(0)}}{dt} = -\frac{1}{(d_\delta)_*\lambda_*}\,[(a_\delta)_*(\lambda_r^{(0)} - \lambda_*) + F\cos\delta_*(\delta_r^{(0)} - \delta_*)],$$

$$\frac{d\lambda_r^{(0)}}{dt} = \frac{1}{(b_\lambda)_*}\,[-(a_\lambda)_*(\lambda_r^{(0)} - \lambda_*) - F\sin\delta_*(\delta_r^{(0)} - \delta_*)].$$

The subscript saterisk denotes the parameters corresponding to the point of rest. The eigenvalues of the Jacobi matrix of the right terms are determined from the quadratic equation

$$\lambda_*(b_\lambda)_*(d_\delta)_*k^2 + k((b_\lambda)_*F\cos\delta_* + \lambda_*(d_\delta)_*(a_\lambda)_*) + (a_\lambda)_*F\cos\delta_* - F\sin\delta_*(a_\delta)_* = 0.$$

In view of the relations $F\cos\delta_* = \lambda_*(c_\lambda)_*$ and $F\sin\delta_* = -\lambda_*(c_\delta)_*$, the solutions of the equation have the form

$$k_{1,2} = \Big\{-(c_\lambda)_*(b_\lambda)_* - (d_\delta)_*(a_\lambda)_*$$
$$\pm\sqrt{[(c_\lambda)_*(b_\lambda)_* + (d_\delta)_*(a_\lambda)_*]^2 - 4(b_\lambda)_*(d_\delta)_*[(a_\lambda)_*(c_\lambda)_* + (c_\delta)_*(a_\delta)_*]}\Big\}\Big/[2(b_\lambda)_*(d_\delta)_*].$$

We note that the coefficients c_λ and a_λ are small enough; therefore, the discriminant is less than zero. These coefficients has little effect on the oscillation frequency but completely determine damping of the oscillations. An increase in the coefficients c_λ and a_λ can be due to both a decrease in the filling of the bearing clearance and a decrease in the rigidity of the elastic liner. The coefficients c_λ and a_λ are equal to zero in the case of complete filling of the clearance and in the absence of a liner.

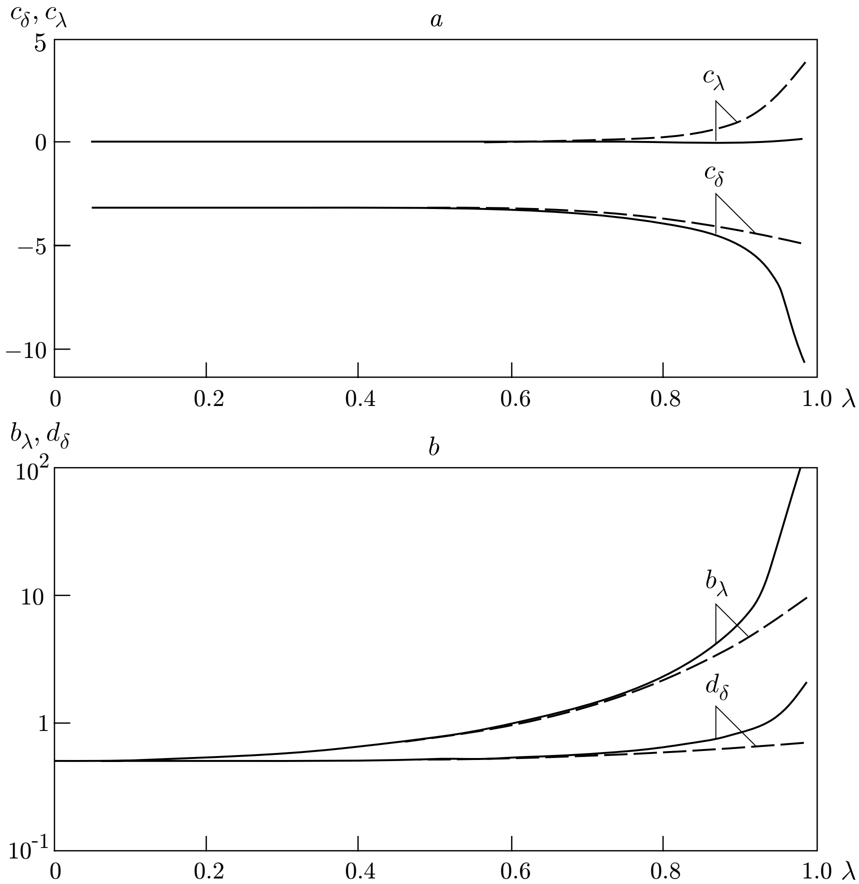

Fig. 2. Rigidity (a) and damping (b) factors versus relative displacement of the shaft: solid curves refer to $\alpha = 0$ and dashed curves refer to $\alpha = 0.005$.

Let us consider equations for the boundary functions. These equations can be obtained by converting to the variable $\tau = t/\varepsilon$ and equating terms of the same order in ε. The zeroth-order boundary functions are equal to zero, and for the first-order functions, we obtain the equations

$$\frac{d^2\delta_s^{(1)}}{d\tau^2} + (d_\lambda)_0 \frac{d\delta_s^{(1)}}{d\tau} = 0, \qquad \frac{d^2\lambda_s^{(1)}}{d\tau^2} + (b_\lambda)_0 \frac{d\lambda_s^{(1)}}{d\tau} = 0.$$

Integrating these equations, we have

$$\frac{d\delta_s^{(1)}}{d\tau} = A \exp\left(-(d_\delta)_0 \tau\right), \qquad \frac{d\lambda_s^{(1)}}{d\tau} = B \exp\left(-(b_\lambda)_0 \tau\right),$$

$$\delta_s^{(1)} = -A \exp\left(-(d_\delta)_0 \tau\right)/(d_\delta)_0, \qquad \lambda_s^{(1)} = -B \exp\left(-(b_\lambda)_0 \tau\right)/(b_\lambda)_0,$$

where the constants A and B are determined from the initial data for the velocities. We note that the perturbations of the velocity components that depend on the boundary functions have the zeroth order in ε.

3. Calculation Results. From results of numerical integration of the Reynolds equation (16), we determine the rigidity factors c_δ and c_λ and the damping factors b_λ and d_δ of the lubricant layer. Curves of these coefficients versus the relative displacement of the shaft are shown in Fig. 2. The rigidity factors are determined as the ratios of the corresponding components of the response of the lubricant layer to the radial displacement of the shaft. The damping factors of the lubricant layer are determined as the constant of proportionality between the radial (azimuthal) perturbation component of the response of the layer and the radial (azimuthal) velocity of the center of the shaft. The absolute values of the rigidity and damping factors increase monotonically with increase in the parameter λ (a decrease in the thickness of the lubricant film). If an elastic liner is absent and the clearance is completely filled, the response of the lubricant layer is perpendicular to the direction of displacement of the shaft [3].

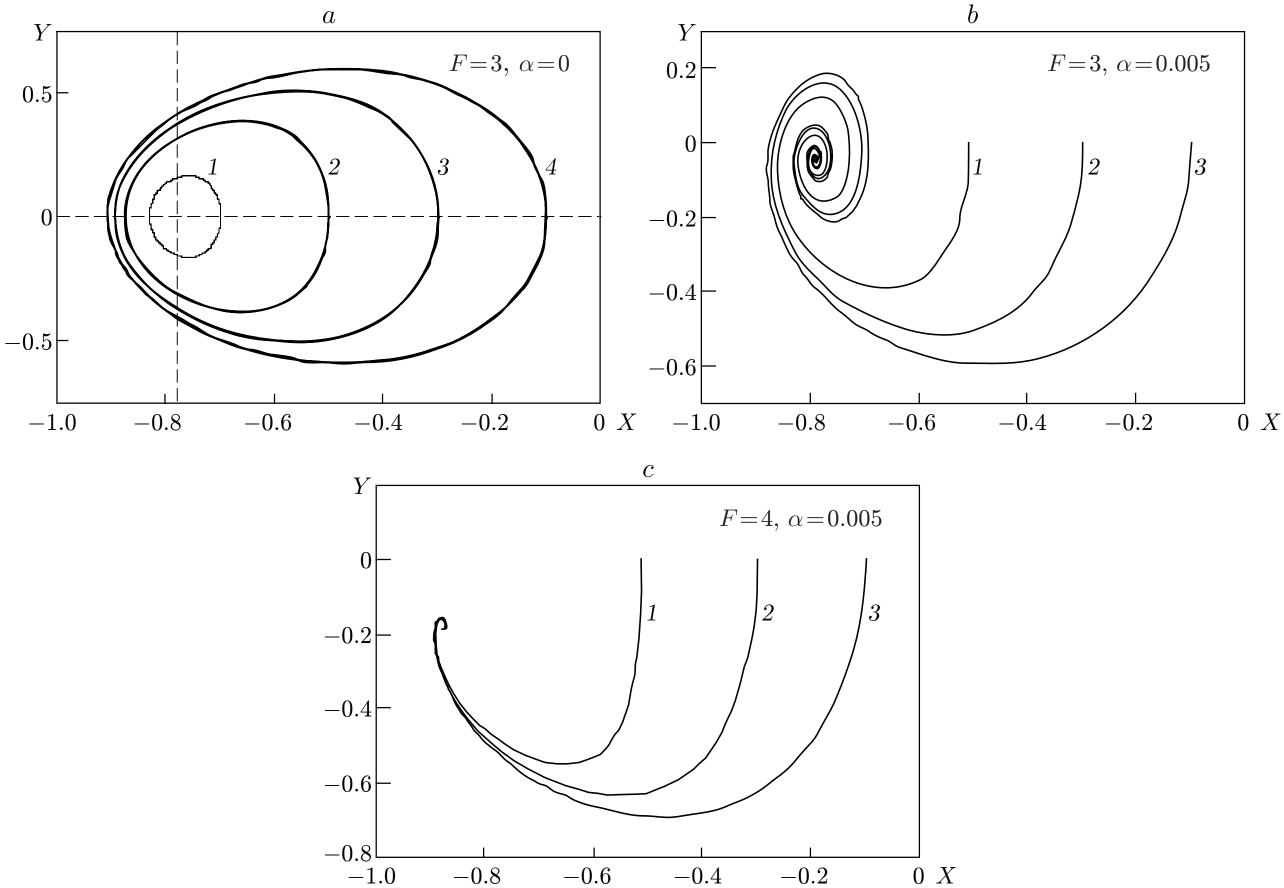

Fig. 3. Motion paths of the shaft without a liner and with a liner under various constant loads: $X_0/\Delta = -0.1$ (1), -0.3 (2), -0.5 (3), and -0.7 (4).

Figure 3 shows the motion paths of the shaft at $Y_0 = 0$ and $X_0/\Delta = -0.1$, -0.3, -0.5, and -0.7. In Fig. 3a, the dimensionless external force is $F = 3$ and the rigidities of the shaft and the bearing are infinite. The values of the force F are normalized by the force $F_0 = 6\mu\omega R_1^3/\Delta^2$. The point of intersection of the dashed lines corresponds to the point of rest. The direction of the external force is opposite the direction of the Y axis. The motion paths of the shaft axis are closed ellipse-like curves corresponding to undamped oscillations of the shaft in the neighborhood of the position of equilibrium. Figure 3b shows three helical paths of the center of the shaft corresponding to different initial conditions in the presence of an elastic liner. The helix pitch increases with increase in the dimensionless parameter α, which is proportional to the ratio of the liner thickness to its rigidity. Transition to the steady state occurs in the regime of damped oscillations. Figure 3c shows the motion paths of the shaft in a bearing with an elastic liner under the same initial conditions as in Fig. 3b but at larger load. It is evident that under this load, the transient process becomes aperiodic.

Figure 4 gives the X coordinate versus time under various initial conditions ($Y_0 = 0$ and $X_0/\Delta = -0.1$, -0.3, and -0.5). In Fig. 4a, the oscillations are nearly harmonic with a dimensionless period $T = 1.2$. In this case, the dimensional period is equal to $T_d = 2.4/\omega$. In Fig. 4b, the oscillations damp rather rapidly. Further increase in the load leads to a sudden increase in the damping factor (Fig. 4c).

Conclusions. The unsteady problem of oscillations of a shaft in a cylindrical plain bearing in the presence of hydrodynamic lubrication and a constant external load. The following results are obtained.

The inertia of the shaft is characterized by the dimensionless small parameter $\varepsilon = m\Delta^3\omega/(6\mu R_1^3) \ll 1$. The effect of this parameter is considerable only at the initial stage, where there is relaxation of the initial velocities to the quasiequilibrium values determined by a balance between the pressure forces of the lubricant layer and the external load. The relaxation time of the initial velocities is $t_\varepsilon \approx \varepsilon/\omega$.

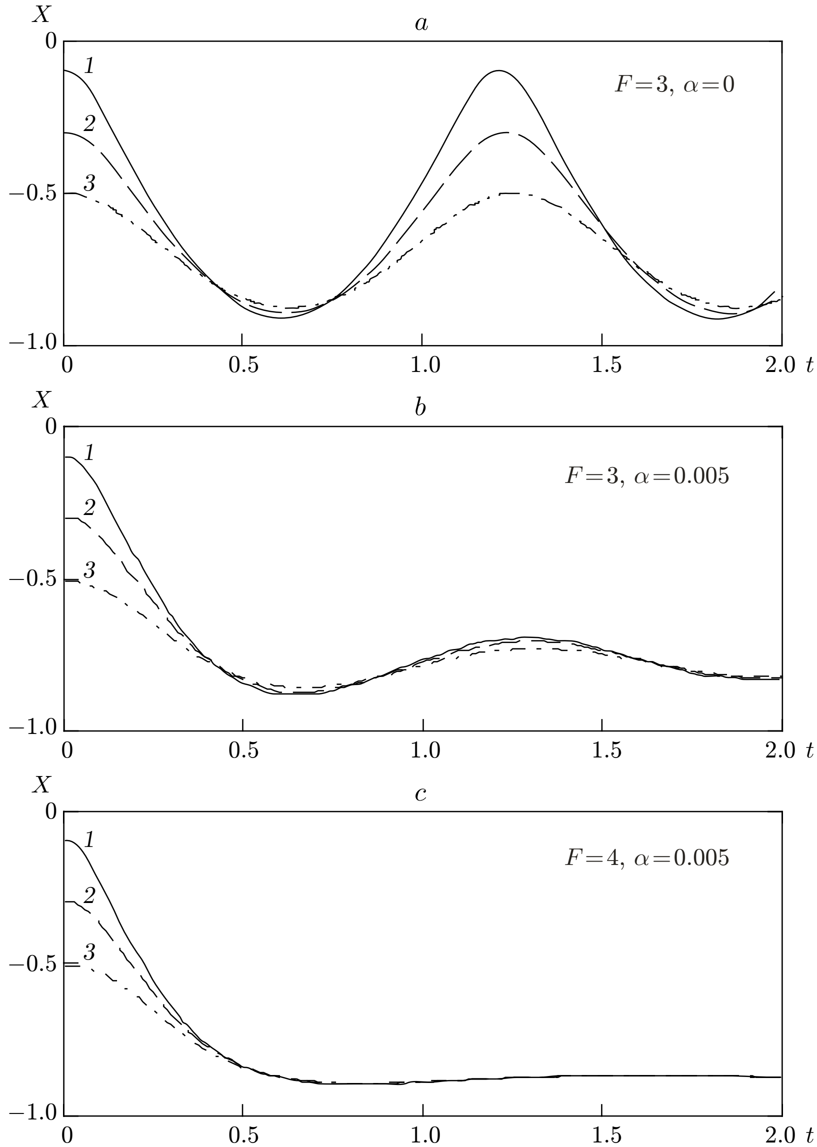

Fig. 4. Displacement of the shaft in the X direction versus time for -0.1 (1), 0.3 (2), and -0.5 (3).

The mode of motions of the center of the shaft described by a system of equations in the inertialess approximation ($\varepsilon \to 0$) is explored. It is shown that under the ideal conditions of rigid surfaces and complete filling of the clearance, the motion of the shaft has an undamped periodic nature. In this case, the center of the shaft moves along a closed path whose shape depends on initial conditions. The oscillation period is $T_d = 2.4/\omega$ for $F = 3$ and increases weakly as the load increases.

In the presence of an elastic liner, the oscillations become damping with the decrement depending on the rigidity factor of the liner. In this case, the motion paths of the center of the shaft becomes helical with the focus at the equilibrium point.

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